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Analysis of self-organized criticality in the Olami-Feder-Christensen model and in real earthquakes

F. Caruso,[1] A. Pluchino,[2] V. Latora,[2] S. Vinciguerra,[3] and A. Rapisarda[2]

[1]*NEST CNR-INFM & Scuola Normale Superiore, Piazza dei Cavalieri 7, I-56126 Pisa and Scuola Superiore di Catania, Università di Catania, Via S. Nullo 5/i, I-95123 Catania, Italy*

[2]*Dipartimento di Fisica e Astronomia, Università di Catania, and INFN sezione di Catania, Via S. Sofia 64, I-95123 Catania, Italy*

[3]*HP-HT Experimental Laboratory of Volcanology and Geophysics, Depertment of Seismology and Tectonophysics, INGV, I-00143 Rome, Italy*

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We perform an analysis on the dissipative Olami-Feder-Christensen model on a small world topology considering avalanche size differences. We show that when criticality appears, the probability density functions (PDFs) for the avalanche size differences at different times have fat tails with a q-Gaussian shape. This behavior does not depend on the time interval adopted and is found also when considering energy differences between real earthquakes. Such a result can be analytically understood if the sizes (released energies) of the avalanches (earthquakes) have no correlations. Our findings support the hypothesis that a self-organized criticality mechanism with long-range interactions is at the origin of seismic events and indicate that it is not possible to predict the magnitude of the next earthquake knowing those of the previous ones.

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In recent years there has been an intense debate on earthquake predictability [1] and a great effort in studying earthquake triggering and interaction [2–5]. Along these lines the possible application of the self-organized criticality (SOC) paradigm [6–14] has been discussed. Earthquakes trigger dynamic and static stress changes. The first acts at short time and spatial scales, involving the brittle upper crust, while the second involves relaxation processes in the asthenosphere and acts at long time and spatial scales [15–21]. In this Rapid Communication, by means of a new analysis, we show that it is possible to reproduce statistical features of earthquakes catalogs [22,23] within a SOC scenario taking into account long-range interactions. We consider the dissipative Olami-Feder-Christensen model [12] on a *small world* topology [24,25] and we show that the probability density functions (PDFs) for the avalanche size differences at different times have fat tails with a q-Gaussian shape [26–29] when finite-size scaling is present. This behavior does not depend on the time interval adopted and is found also when considering energy differences between real earthquakes. It is possible to explain this result analytically assuming the absence of correlations among the sizes (released energies) of the avalanches (earthquakes). This finding does not allow one to predict the magnitude of the next earthquake knowing those of the previous ones.

The Olami-Feder-Christensen (OFC) model [12] is one of the most interesting models displaying self-organized criticality. Despite its simplicity, it exhibits a rich phenomenology resembling real seismicity, such as the presence of aftershocks and foreshocks [14]. In its original version the OFC model consists of a two-dimensional square lattice of $N = L^2$ sites, each one connected to its four nearest neighbors and carrying a seismogenic force represented by a real variable F_i, which initially takes a random value in the interval $(0, F_{th})$. In order to mimic a uniform tectonic loading all the forces are increased simultaneously and uniformly, until one of them reaches the threshold value F_{th} and becomes un-

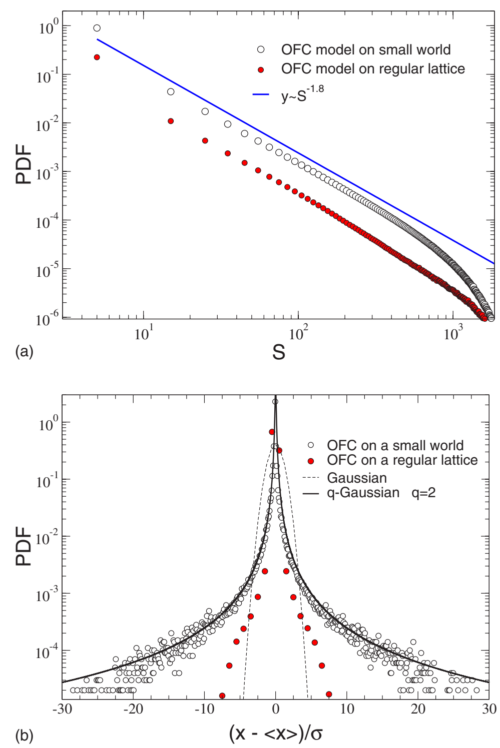

FIG. 1. (Color online) (a) Power-law distribution of the avalanche size S for the OFC model ($\alpha=0.21$) on a small world topology (open circles) and on a regular lattice 64×64 (full circles). Data were shifted for clarity. A fitting curve with slope $\tau=1.8$ is also reported as full line. (b) PDF of the avalanche size differences (returns) $x(t)=S(t+1)-S(t)$ for the OFC model on a small world topology (critical state: open circles) and on a regular lattice (noncritical state: full circles). Returns are normalized to the standard deviation σ. The first curve has been fitted with a q-Gaussian (full line) with an exponent $q\sim 2.0\pm 0.1$. A standard Gaussian (dashed line) is also reported for comparison. All the curves were normalized so as to have unitary area. See the text for further details.

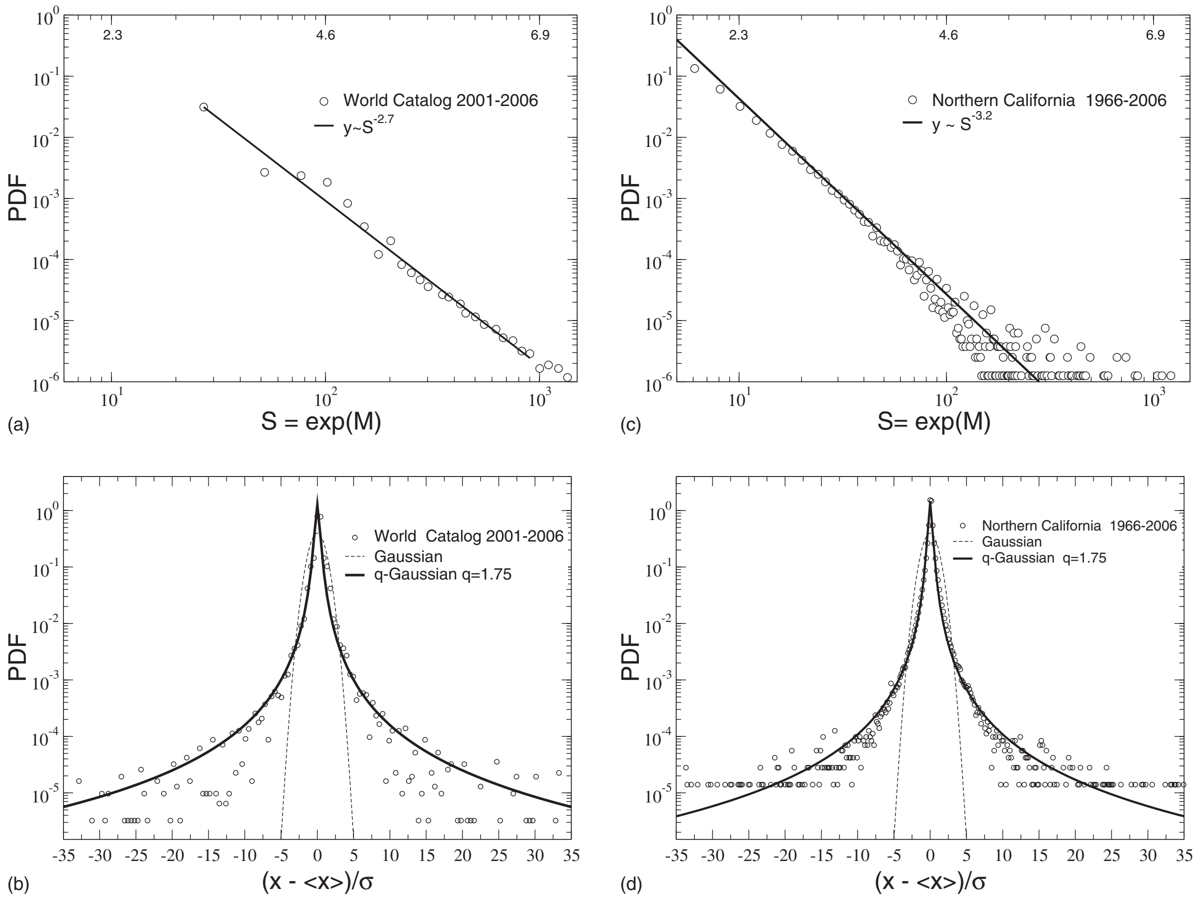

FIG. 2. Power-law distribution of $S=\exp(M)$, M being the magnitude for the world catalog (a) and for the northern California catalog (c). The correspondent power-law fits are also reported. We show the corresponding values of the magnitude M in the upper part of the figures. PDFs of the energy differences $x(t)=S(t+1)-S(t)$ are shown in (b) for the worldwide seismic catalog and in (d) for the northern California catalog. In both the figures the data have been fitted with a q-Gaussian (full line) with $q\sim 1.75\pm 0.15$. A standard Gaussian is plotted as a dotted line in all the figures for comparison. See the text for further details.

stable ($F_i \geqslant F_{th}$). The driving is then stopped and an "earthquake" (or avalanche) starts;

$$F_i \geqslant F_{th} \Rightarrow \begin{cases} F_i \to 0, \\ F_{nn} \to F_{nn} + \alpha F_i, \end{cases} \qquad (1)$$

where "nn" denotes the set of nearest-neighbor sites of i. The number of topplings during an avalanche defines its size S, while the dissipation level of the dynamics is controlled by the parameter α. The model is conservative if $\alpha=0.25$, while it is dissipative for $\alpha<0.25$. In the present Rapid Communication we consider the dissipative version of the OFC model with $\alpha=0.21$ [30], on a regular lattice with $L=64$ and open boundary conditions (i.e., we impose $F=0$ on the boundary sites). But, in order to improve the model in a more realistic way, we introduce a small fraction of long-range links in the lattice so as to obtain a small world topology [25]. Just a few long-range edges create shortcuts that connect sites which otherwise would be much further apart. This kind of structure allows the system to synchronize and to show both finite-size scaling and universal exponents [24]. The curves obtained for different sizes of the system collapse into a single one. Furthermore, a small world topology is expected to model more accurately earthquakes spatial correlations, taking into account long-range as well as short-range seismic effects [2–5]. In our version of the OFC model the links of the lattice are rewired at random with a probability p as in the one-dimensional model of Ref. [25]. In [24] it was shown that the transition to obtain small world features and criticality is observed at $p=0.02$.

In Fig. 1(a) we plot the distribution of the avalanche size time-series $S(t)$ for the OFC model on a small world topology (open circles) and on a regular lattice (full circles). In our case the time t is a progressive discrete index labeling successive events and is analogous to the "natural time" successfully used in [21]. We have considered up to 10^9 avalanches to have good statistics. In both cases the data follow a power-law decay $y\sim S^{-\tau}$ with a slope $\tau=1.8\pm 0.1$ even if criticality is present only for the small world topology [24]. In recent years SOC models have been intensively studied considering time intervals between avalanches in the critical regime [19]. Here we follow a different approach which reveals interesting information on the eventual criticality of the model under examination. Inspired by recent studies on turbulence and intermittent data [27–29], we focus our attention on the "*returns*" $x(t)=S(t+\Delta)-S(t)$, i.e., on the differences between avalanche sizes calculated at time $t+\Delta$ and at time t, Δ being a discrete time interval.

The resulting signal is extremely intermittent at criticality, since successive events can have very different sizes. On the other hand, if the system is not in a critical state this intermittency character is very reduced. In Fig. 1(b) we plot as open circles the probability density function (PDF) of the returns $x(t)$ (with $\Delta=1$) obtained for the critical OFC model on small world topology. The returns are normalized in order to have zero mean and unitary variance. The curves reported have also unitary area. A behavior very different from a Gaussian shape (plotted as a dashed curve) is observed. Data are very peaked with fat tails. On the other hand, for the model on regular lattice, even if power laws are found, the model is not critical since no finite-size scaling is observed [24]. In this case no fat tails exist, although a sensible departure from Gaussian behavior is still present (see full circles). These findings suggest a new powerful way for characterizing the presence of criticality. They are also reinforced by similar results on other SOC models not reported here for lack of space. Another remarkable feature is that such a behavior does not depend on the interval Δ considered for the avalanche size difference. Also reshuffling the data, i.e., changing in a random way the time order of the avalanches, no change in the PDFs is observed. The data reported in Fig. 1(b) for the critical OFC model on a small world can be well fitted by a *q-Gaussian* curve $f(x)=A[1-(1-q)x^2/B]^{1/(1-q)}$ typical of Tsallis statistics [26]. This function generalizes the standard Gaussian curve, depending on the parameters A,B and on the exponent q. For $q=1$ the normal distribution is obtained again, so $q\neq 1$ indicates a departure from Gaussian statistics. The q-Gaussian curve, reported as a full line, reproduces very well the model behavior in the critical regime, yielding in our case a value of $q=2.0\pm0.1$. In order to compare these theoretical results with real data sets, we repeated the previous analysis for the worldwide seismic catalog available online [22]. We considered 689 000 earthquakes in the period 2001–2006. As a further term of comparison, we selected a more complete seismic data set, i.e., the Northern California catalog for the period 1966–2006 [23]. The latter is a very extensive seismic data set on one of the most active and studied faults on the Earth, i.e., the San Andreas Fault. In this case the total number of earthquakes is almost 400 000. As pointed out by several authors [14] the energy, and not the magnitude, is the quantity which should be considered equivalent to the avalanche size in the OFC model. In this paper we consider the quantity $S=\exp(M)$, M being the magnitude of a real earthquake. This quantity is simply related to the energy dissipated in an earthquake, the latter being an increasing exponential function of the magnitude.

In Figs. 2(a) and 2(c) we plot the PDFs of S for the world catalog and the Northern California catalog. Power-law decays with exponents $\tau=2.7\pm0.2$ and $\tau=3.2\pm0.2$ reproduce the PDFs for the two cases, respectively. Then we consider the PDFs of the corresponding returns $x(t)=S(t+\Delta)-S(t)$ (with $\Delta=1$) and we plot them in Figs. 2(b) and 2(d). Also for real data t is a progressive discrete index labeling successive events. As for the critical OFC model previously discussed, fat tails and non-Gaussian probability density functions are observed. In both cases the experimental points can be fitted

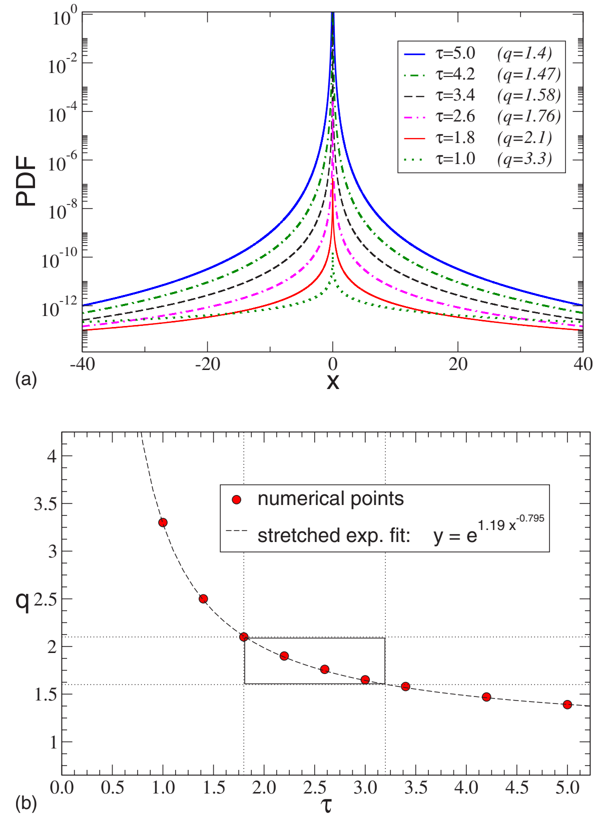

FIG. 3. (Color online) (a) We show the curves obtained by calculating the integral of Eq. (3) considering different values of τ. (b) We plot the relationship between the values of q and τ (full circles), where q is obtained by fitting the curves in (a) with a q-Gaussian. The resulting points can be very well reproduced by a stretched exponential, $y=e^{1.19x^{-0.795}}$, drawn as a dashed line. Inside the plotted box, one can find the values of τ and q found for the OFC model and the real data sets.

by a q-Gaussian curve, obtaining an exponent $q=1.75\pm0.15$, a value which is compatible, within the errors, to that one found for the OFC model. The world catalog presents large fluctuations in the tails due to the significant incompleteness given from the lack of small magnitudes events at the global scale. Also for the real earthquakes data, by changing the interval Δ of the energy returns x, or by reshuffling the time-series $S(t)$, no change in the PDFs is observed. In general, both for the OFC and for the real earthquakes catalogs the avalanche sizes (energies) S occur with a power-law probability $p(S)\sim S^{-\tau}$. In the hypothesis of no correlation between the size of two events, the probability of obtaining the difference $x=S'(t+\Delta)-S(t)$ (whatever Δ) is given by

$$P(x) = K\int_0^\infty dS\int_0^\infty dS'(SS')^{-\tau}\delta(S'-S-x)$$

$$= K\int_\epsilon^\infty dS[S(S+|x|)]^{-\tau}, \qquad (2)$$

where K is a normalization factor. The absolute value $|x|$ takes into account the exchange of S' with S. The integral is divergent for $S=0$, so we consider a small positive value ϵ as an inferior limit of integration. Then one gets

$$P(x) = K\frac{\epsilon^{-(2\tau-1)}}{2\tau-1}\,{}_2F_1\left(\tau,2\tau-1;2\tau;-\frac{|x|}{\epsilon}\right), \tag{3}$$

${}_2F_1$ being the confluent hypergeometric function. The probability density function (3) is plotted for various values of τ in Fig. 3(a). All these curves can be very well reproduced by means of q-Gaussians, whose values of q do not depend on ϵ (since we have verified that the latter changes only the normalization factor). The relation between q and τ is shown in Fig. 3(b), where the points are well fitted by a stretched exponential curve. Notice that increasing τ, i.e., when the power law tends to an exponential, q tends to 1 as expected. The value we get for the avalanche size power law of the OFC model with a small world topology is $\tau=1.8$, which corresponds, according to Fig. 3, to a value of $q\sim 2.1$ in agreement with the curve shown in Fig. 1(b) within the errors. A similar correspondence can be found for the returns of real earthquakes data. In particular, for $\tau=2.7\pm 0.2$ and $\tau=3.2\pm 0.2$, see Figs. 2(a)–2(c), one gets values of q compatible, inside the errors, with the value $q\sim 1.75$ found in Figs. 2(b)–2(d); see the values inside the box of Fig. 3(b). This result explains the q-Gaussian fat-tails in terms of differences between uncorrelated (in time) avalanches (earthquakes) sizes. On the other hand, we have checked that when avalanches are generated by a deterministic chaotic dynamics, one finds a dependence on the interval considered for the size returns and the resulting PDFs cannot be explained with the function (3).

In conclusion we have presented a new analysis which is able to discriminate in a quantitative way real SOC dynamics. The results here presented for the OFC model and earthquakes data, on one hand give further support to the hypothesis that seismicity can be explained within a dissipative self-organized criticality scenario when long-range interactions are considered. On the other hand, although temporal and spatial correlations among avalanches (earthquakes) do surely exist and a certain degree of statistical predictability is likely possible, they indicate that it is not possible to predict the magnitude of seismic events.

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